Sub-optimal Decoding of Block and Lattice Codes

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Abstract

We present several bounded distance decoding algorithms for a family of block codes. These algorithms can be efficiently employed for decoding lattice codes. A thorough investigation of the performance and decision regions of the proposed algorithms are performed. These suggest that there are three distinct types of nearest neighbors classified according to their affect on the decision region.

1 Introduction

The problem of optimal decoding of block and lattice codes in additive white Gaussian noise channels is very interesting and has been studied intensively along the years. In practice, however, one is usually willing to sacrifice some performance, or in other words optimality, for decoding complexity reduction. The motivation is to reduce decoding complexity as much as possible while paying as little as possible in performance. **Decoding complexity** is traditionally measured in terms of the amount of real-number operations (rather than algebraic operations) as they primarily determine decoding time and/or the hardware implementation size. Note, however, that when the real-number decoding complexity decreases considerably, the algebraic decoding complexity can no longer be neglected [1].

1.1 sub-optimal decoding

The most efficient decoder in terms of real-number operations is obviously the algebraic hard-decision decoder. The problem is that its performance are 2-3dB (depending on the signal to noise power ratio) worse than the optimal decoder. Bounded-distance decoders, on the other hand, present a much more promising tradeoffs between complexity and performance. By bounded-distance decoding we mean that the decoder guarantees correct decoding at least up to half the minimum *Euclidean* distance of the code (rather than half the minimum *Hamming* distance as in the case of algebraic hard-decision decoding).

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2 Summary

In this work we present several decoding algorithms, and thoroughly investigate their performance and decision regions.

2.1 Decoding algorithms

We present several soft decision bounded-distance decoding algorithms for a family of linear *q-ary* block codes [2]. These decoders have different complexity/performance tradeoffs making it possible to choose the most appropriate decoder for a given scenario. The general idea behind these algorithms is to divide the received hard-decision vector into two (or more) non-overlapping, or in the more general case partially overlapping, blocks of symbols, such that the symbols in each block constitute an information vector. These algorithms can be efficiently employed for decoding interesting and practical lattice codes such as the Leech lattice (see [1] for more details).

2.2 Performance evaluation

We evaluate the performance of the proposed algorithms both analytically and by means of computer simulations. In the analytical analysis we carefully investigate the decision regions of the presented decoders in real-space. This is interesting because with bounded distance decoding (as compared to optimal decoding), performance degradation is dominated by the behavior of the decoder outside the (bounded-distance) hyper-spheres whose centers are the codewords. We reveal some interesting phenomena, sug-

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gesting that there are three distinct types of nearest neighbors classified according to their affect on the decision region. These include:

- codewords
- non-codewords that are generated in the decoding process
- what we call *pseudo nearest neighbors.*

In particular, we show that some of the proposed algorithms realize a significant portion of the optimal decision region, i.e. the *Voronoi region*, in the vicinity of the bounded distance hyper-sphere, and thus perform close to optimum.

To get a complete picture of the algorithms behavior, we simulated several block codes over a wide range of signal to noise power ratios. A simulation of several versions of GMD (generalized minimum distance) decoding [3] has also been performed for comparison. Figures 1 and 2 present the probability of word error for the binary $(8,4,4)$ - extended Hamming code, and the $(8,4,4)$ code over $GF(4)$, respectively. AB1, AB12, and AB2 are three of the proposed algorithms; GMD1, GMD2, and GMD3 are three versions of G-MD decoding. We also include the optimal soft decision decoder, denoted by OPT, and the optimal hard decision decoder, denoted by HD. AB1 has the best performance among the presented decoders, note how close it is to the optimum. The simulation results verify our analytical derivations.

Finally, we present some interesting images that vividly exhibit the different phenomena mentioned above. These computer images show the decision region of a specific codeword in various informative 2-dimensional cross sections (of the real space of the code). For example, Figure 3 depicts the decision region of the codeword $c_o = 00000000 \in (8,4,4)$ – extended binary Hamming code. The cross section displayed is the plane defined by the three points: c_o and c_1 which are both codewords; and p_2 which is a nearest neighbor but not a codeword. The dotted line represents the border of the decision region of c_o for optimal decoding; the solid line represents the border of the decision region of c_o for one of the proposed algorithms; and the dash-dotted line is the cross section of the hyper-sphere of radius $\frac{1}{2}d_o$ around c_o. Note that the affect of p_2 (which is a nearest neighbor of the algorithm, but not a codeword) on the decision region of c_o is quite different then that of c_1 (which is a nearest neighbor and a codeword).

Acknowledgments

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References

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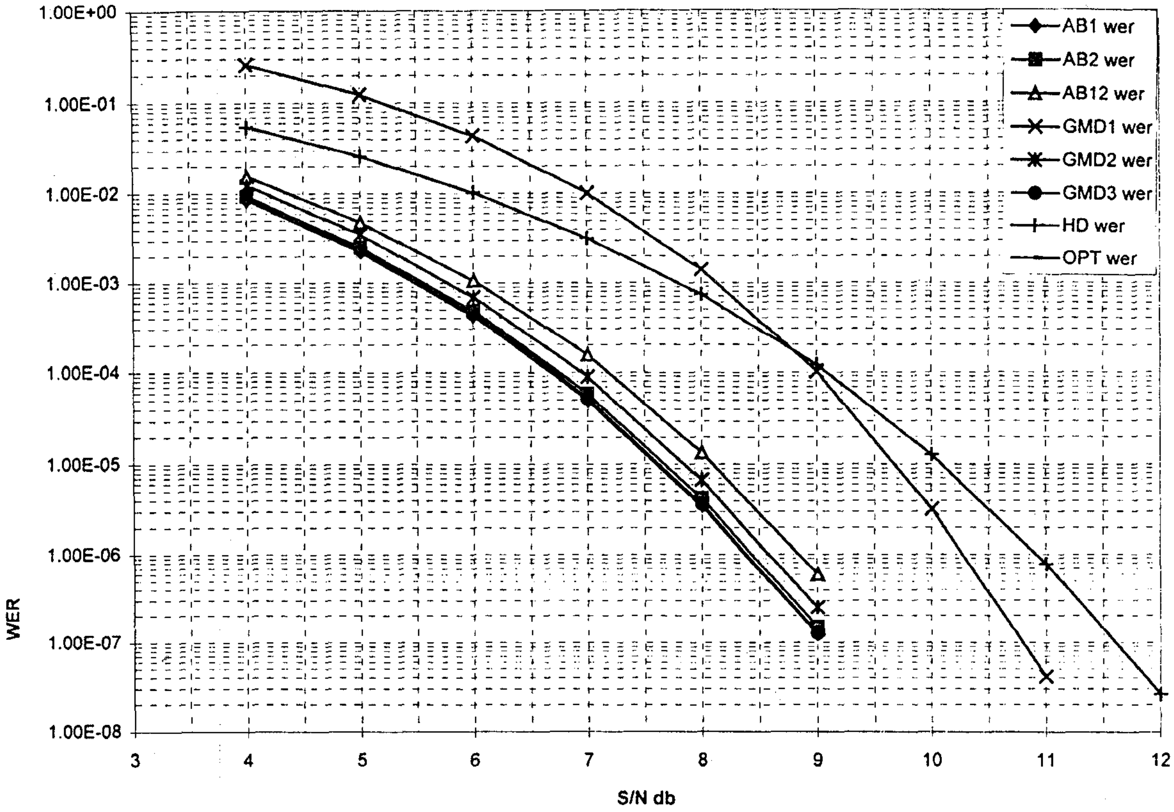

Figure 1: Binary (8,4,4) extended Hamming code

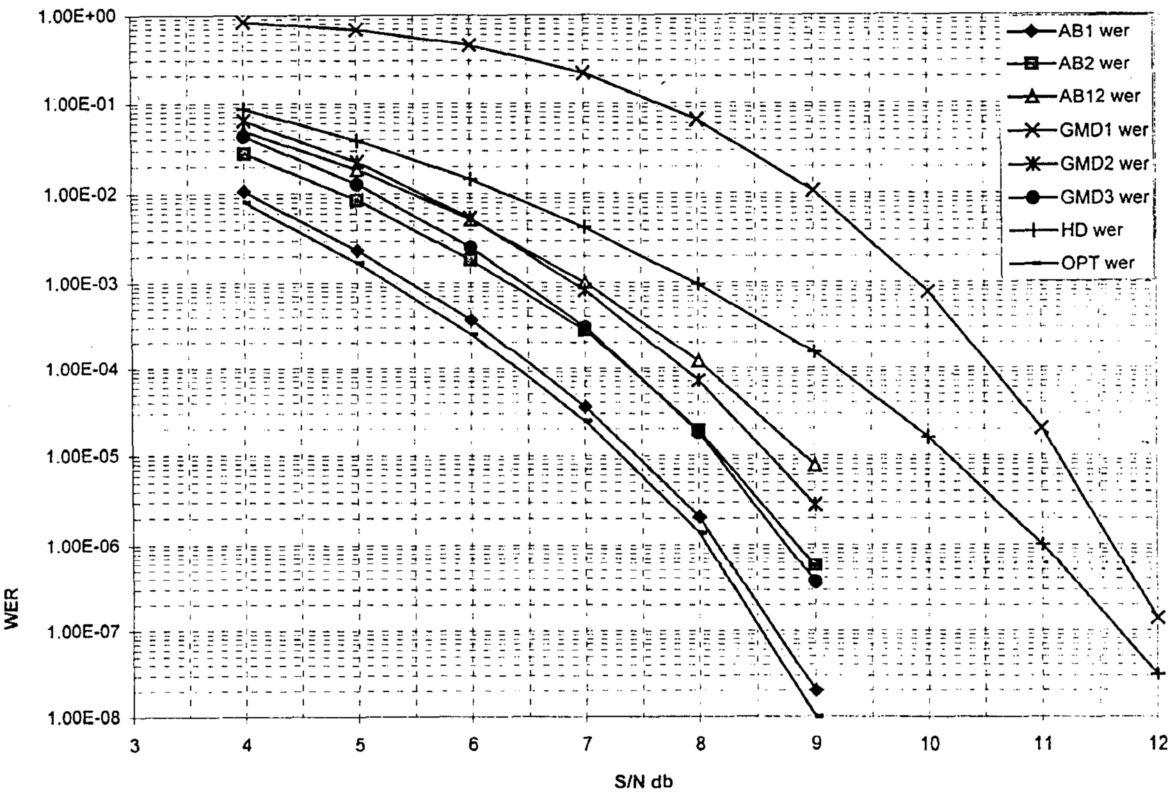

Figure 2: (8,4,4) code over $GF(4)$

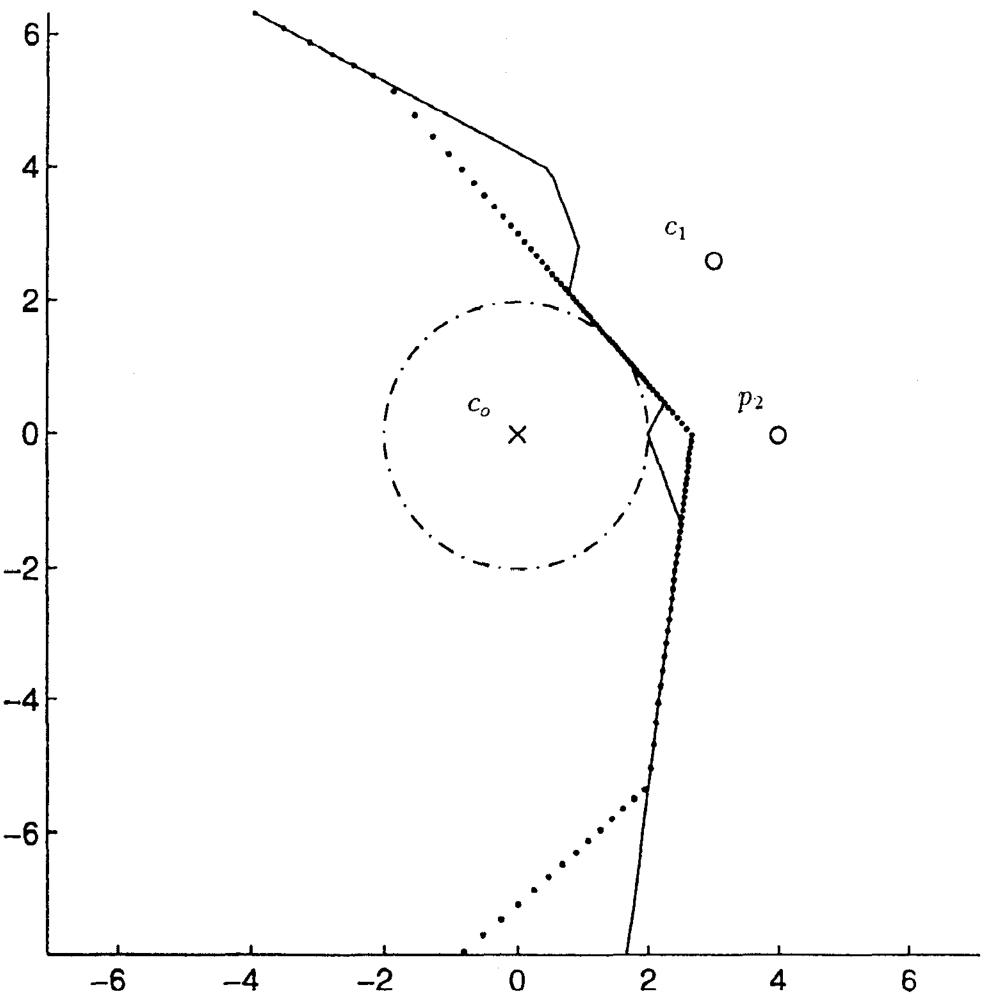

Figure 3: A cross section of the decision region of c_o